Microeconomic Theory II

Spring 2026

Midterm Exam

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Solutions

Quick, Incomplete, and not Guaranteed

Question 1. Consider the following normal-form game between Player 1 (rows) and Player 2 (columns):

		Player 2			
		A	B	C	D
Player 1	M	$3, 3$	$7, 2$	$3, 7$	$1, 1$
	N	$2, 7$	$1, 2$	$0, 3$	$7, 1$
	O	$7, 1$	$3, 6$	$7, 1$	$3, 6$
	P	$4, 0$	$4, 8$	$3, 0$	$2, 8$

(a) Apply iterated deletion of strictly dominated strategies (IDSDS). For each strategy deleted, clearly state which strategy dominates it. For each remaining strategy, demonstrate or explain why it is not strictly dominated.

Delete P; no further deletions.

P is strictly dominated by (for example) $\frac{1}{3}M + \frac{2}{3}O$. No further deletions. Since each remaining strategy is a best reply to some pure opponent strategy (M to B or C; N to D; O to A or C; A to N; B to O; C to M; D to O), none is strictly dominated.

Note 1a.1: Several students deleted D after deleting P, on the grounds that $B \geq D$ in every column. IDSDS requires *strict* dominance. D is weakly dominated by B but it is a best reply to O. Some students similarly deleted D because it is ‘‘never a unique best reply.’’ Uniqueness is not the criterion: in two-player games, a strategy is strictly dominated if and only if it is *never* a best reply at all, whether uniquely or tied.

Note 1a.2: Some students checked whether any *pure* strategy of Player 1 dominates P, found none, and concluded P is not strictly dominated. This is insufficient: a strategy can be strictly dominated by a *mixed* strategy even when no pure strategy dominates it.

Note 1a.3: Conversely, some argued P is strictly dominated because it is not a best reply to any *pure* strategy of Player 2, without checking whether P might be a best reply to a mixed strategy. In a two-player game, a strategy is strictly dominated if and only if it is never a best reply to *any* (pure *or* mixed) belief over the opponent's strategies. It is reasonable to select potential candidates for dominated strategies by checking pure strategies.

Note 1a.4: The strategy $\frac{1}{2}M+\frac{1}{2}O$ does not strictly dominate P. This mixture only *weakly* dominates P: against column D, $\frac{1}{2}(1)+\frac{1}{2}(3) = 2 = u_1(P, D)$, so there is a tie. For strict domination, every column must yield a strict inequality; at D this requires $\alpha + (3)(1 - \alpha) > 2$, i.e., $\alpha < \frac{1}{2}$.

Note 1a.5: Some students found a strategy to be weakly dominated and then additionally searched for a strategy that strictly dominates it, as if weak dominance required a follow-up check. Once a strategy is shown to be a best reply to *anything*---any pure or mixed strategy of the opponent, uniquely or tied---it is immediately established as not strictly dominated. No further search is needed.

(b) Demonstrate or argue that the game does not have an equilibrium in which Player 1 plays N with positive probability.

The easiest demonstration proceeds in two steps: (i) show that if N is played with positive probability, Player 2 will not play D, and (ii) show that if Player 2 does not play D, Player 1 will not play N, leading to a contradiction.
For step (i), note that B weakly dominates D. Compare Player 2's payoffs from B and D:

$$u_2(B) - u_2(D) = (2\sigma_M + 2\sigma_N + 6\sigma_O) - (1\sigma_M + 1\sigma_N + 6\sigma_O) = \sigma_M + \sigma_N.$$

Thus, if $\sigma_N > 0$, B does better than D. Therefore, $\sigma_D = 0$.
For step (ii), note that Player 1 strictly prefers O to N for any Player 2 strategy in which $\sigma_D = 0$, contradicting $\sigma_N > 0$.

Note 1.b1: The argument in part (b) is frequently misread as a domination argument. Strict domination is a property of the payoff matrix: strategy s is strictly dominated if some other strategy yields a strictly higher payoff for *every* opponent strategy, independent of any assumption about play.

The argument in part (b) is categorically different: it *assumes* $\sigma_N > 0$ and shows that *under this assumption* Player 2 strictly prefers B to D. This is a conditional payoff comparison in a proof by contradiction. D is not strictly dominated---it is a best reply to O. Treating $\sigma_D = 0$ as if it followed from domination, and carrying the deletion of D into other parts of the problem, is an error.

Note 1b.2: Related to the above: some students argued that D is weakly dominated by B and therefore cannot be played in any Nash equilibrium. This is incorrect. Weak dominance does not prevent a strategy from appearing in a NE. Question 2 provides a direct example: $(A, C;\ R, X)$ is a Nash equilibrium even though R, X is weakly dominated by R, Y. What part (b) actually establishes is not that D is dominated, but that B strictly beats D *conditional on* $\sigma_N > 0$---a specific and very different claim. Similarly, we are not "deleting" N after ruling out D, we are showing that it is never a best reply conditional on $\sigma_D = 0$.

Note 1b.3: Some students showed that Player 2 can be indifferent between B and D only when $\sigma_N = 0$, and concluded the argument. This rules out only equilibria in which Player 2 uses *both* B and D; equilibria in which Player 2 uses only A, B, C or only A, C, D are unaddressed. The full argument must also establish $\sigma_D = 0$ via the strictly higher payoffs from B.

Note 1b.4: Showing that there is no *pure-strategy* Nash equilibrium with player 1 playing N is insufficient. The question asks to rule out $\sigma_N > 0$ in *any* equilibrium, including mixed ones. Similarly, after correctly establishing $\sigma_D = 0$, it is not enough to check that N is not a best reply to any *pure* strategy in $\{A, B, C\}$. Player 2 may use any mixture over $\{A, B, C\}$, and N must be ruled out against all such mixtures.

Note 1b.5: Some students argued that a mixed-strategy NE requires Player 2 to be indifferent among all four of Player 2's strategies, that this cannot hold when N is in Player 1's support, and therefore N is ruled out. In fact, it is impossible in this game to make Player 2 indifferent between all strategies, which means that the same argument could be used to rule out *every* strategy! A player must be indifferent only among strategies *in the equilibrium support*. The support must be determined first. This argument eliminates only equilibria in which Player 2 mixes over all four strategies; equilibria in which Player 2's support is a strict subset are unaddressed.

(c) Find all Nash equilibria. Demonstrate that each is an equilibrium, and that no others exist.

$\boxed{\text{Unique NE: } (\tfrac{1}{2}M + \tfrac{1}{2}O,\ \tfrac{1}{2}B + \tfrac{1}{2}C) \text{, with payoffs } (5,4).}$

From Part (a), we can eliminate P, thus $\sigma_P = 0$. From Part (b), we have $\sigma_N = 0$. Thus, player 1 can use only M and O.

Assume first that $\sigma_M > 0, \sigma_O > 0$. Then Player 2 will never play D as B is always better: $u_2(B) - u_2(D) = \sigma_M > 0$. Player 2 will also never play A because C is always better: $u_2(C) - u_2(A) = 4\sigma_M > 0$.

Once we recognize that only strategies M, O, B, C may receive positive probability, we find the mixed-strategy equilibrium.
Player 1's indifference between M and O implies:

$$u_1(M) = 7\sigma_B + 3\sigma_C = u_1(O) = 3\sigma_B + 7\sigma_C$$

With $\sigma_B + \sigma_C = 1$, we get $\sigma_B = \sigma_C = \frac{1}{2}$.
Player 2's indifference between B and C implies:

$$u_2(B) = 2\sigma_M + 6\sigma_O = u_2(C) = 7\sigma_M + \sigma_O.$$

Which gives $\sigma_M = \sigma_O = \frac{1}{2}$. We can confirm that the strategies not in the support all do worse. For example, consider player 2. In equilibrium, $u_2(B) = u_2(C) = 4$. Against Player 1's equilibrium strategy, $u_2(A) = 2 < 4$ and $u_2(D) = 3.5 < 4$.

We are *almost* done. We assumed above that $\sigma_M > 0, \sigma_O > 0$ and thus have not ruled out equilibria in which Player 1 plays only M or only O. We can verify by inspection that no pure-strategy NE exist, but this still allows for Player 1 to play a pure strategy and for Player 2 to use a mixed strategy. First, assume player 1 plays M. The only best response is C, but M is not a best response to C. Next assume player 1 plays O. Then player 2 is indifferent between B and D and thus any mixture of B and D is a best reply. However, if player 2 plays some mixture of B and D, player 1 earns 3 from O but earns 4 from $\frac{1}{2}M + \frac{1}{2}N$, so O is not a best response.

Note 1c.1: Since we determined from (a) and (b) that P is dominated and N is not used in any equilibrium, there is no need to assign them (positive) probability or attempt to make players indifferent between them and other strategies when calculating a candidate equilibrium.

Note 1c.2: Some students assumed an equilibrium support (e.g., $\{M, O\} \times \{A, B\}$), found probabilities making each player indifferent within the support, and reported this as a NE. Verifying within-support indifference is necessary but not sufficient: every strategy *outside* the support must also yield a weakly lower payoff. In fact, Player 2 would earn more from C, and thus would deviate; this is not a NE. Similarly, even calculating the correct equilibrium with support $\{M, O\} \times \{B, C\}$ requires confirming that the payoffs are higher than strategies outside the support unless they were eliminated by IDSDS.

Note 1c.3: An answer like $(\sigma_M, \sigma_N, \sigma_O)$ and $(\sigma_A, \sigma_B, \sigma_C, \sigma_D)$, without deriving the values, is notation, not an answer. The question asks to *find* all Nash equilibria, which requires deriving specific probabilities, verifying mutual best replies, and showing no other equilibria exist.

Note 1c.4: Some students used Player 1's own payoffs to derive Player 1's mixing probabilities σ. In a mixed-strategy NE, each player's mixing probabilities are chosen to make the *other* player indifferent. Player 1's σ makes Player 2 indifferent; Player 2's makes Player 1 indifferent. Using a player's own payoffs gives conditions on the *opponent's* strategy, not on that player's own.

Note 1c.5: In equilibrium, a player must be indifferent between all of the strategies he uses in equilibrium. That is, Player 1 must receive the same utility from playing M as from playing O. Some confused the logic and tried to make Player 1 indifferent between Player 2's strategies, B and C, effectively saying that Player 1 would be indifferent to what Player 2 does. That is not the right condition as Player 1 does not control Player 2's choices. This logic also would allow us to mix using dominated strategies.

Note 1c.6: You can't use the result of part (b) that $\sigma_N = 0$ in any NE as grounds to "delete" N before solving part (c). Only being strictly dominated allows for "deletion". A strategy's absence from the equilibrium support does not mean it is dominated and can be deleted. This logic is circular: the result of part (b) is a consequence of the equilibrium analysis, not a premise that can be used to simplify it. In fact, if N were deleted, the game would have a pure-strategy ‘‘equilibrium’’ (O, D), but this is clearly wrong as against D, Player 1's best reply is N, not O.

(d) Suppose this game is repeated twice. Does there exist a subgame-perfect Nash equilibrium in which players play pure strategies in the first period? Explain.

$\boxed{\texttt{No.}}$

The stage game has a unique NE---the mixed strategy from part (c). We know that if a stage-game has a unique NE, then the only SPNE of the *finitely* repeated game is playing the stage game NE at every stage. Thus, the unique SPNE will involve only the mixed strategy.
To review the logic: In any SPNE of the two-period game, the second period must be played according to a stage-game NE. Since there is a unique NE (in mixed strategies), second-period payoffs are fixed at $(5, 4)$ regardless of first-period play. Since the second-period payoffs do not vary with first-period actions, adding a constant to payoffs does not change best responses. Therefore, the first-period game is strategically identical to the one-shot stage game, which has no pure NE. Hence no pure-strategy first-period SPNE exists.

Note 1d.1: Some students concluded that because the stage game has no pure-strategy NE, the repeated game cannot have a pure-strategy SPNE. This is not generally true. With *multiple* stage-game NE, the threat of switching to a worse continuation can sustain first-period play that is not a stage-game equilibrium. The relevant feature here is that the stage game has a *unique* NE: every first-period history leads to the same continuation value, leaving no differential for reward or punishment. The impossibility follows from uniqueness of the stage-game NE, not from the absence of a pure-stage-game NE.

Note 1d.2: Some students answered yes on the grounds that the stage game has more than one Nash equilibrium. Even if this were true, multiple NE are necessary but not sufficient to sustain a specific first-period profile. To work, second-period payoffs following different first-period outcomes must differ enough to change the best replies. Thus, even if the stage game had multiple equilibria, this does not complete the argument.

Note 1d.3: There was some confusion between NE of the stage game and of the repeated game. SPNE are always a subset of NE, but that means that SPNE of the repeated game are a subset of the NE of the repeated game. It does not mean that the SPNE of the repeated game are somehow a subset of stage-game NE; in fact, these are two very different objects (a strategy `in the stage game is a single action. A strategy in the repeated game includes contingencies for all 16 possible outcomes in the second period).`

(e) Suppose this game is infinitely repeated with common discount factor $\delta < 1$. Does there exist a subgame-perfect Nash equilibrium in which players play (N, A) on the equilibrium path? Carefully explain.

$\boxed{\texttt{No.}}$

The equilibrium payoff from part (c) is 5 for Player 1. The path (N, A) gives Player 1 a per-period payoff of $u_1(N, A) = 2$. The best deviation for Player 1 against A is to play O, yielding $u_1(O, A) = 7$.

The root problem is that the NE payoff $5 > 2 = u_1(N, A)$: reverting to the NE *rewards* rather than punishes a deviation. A credible equilibrium reversion punishment can't make (N, A) worth it, regardless of δ.

To see this algebraically: for a grim-trigger strategy (revert to NE forever after any deviation), Player 1's constraint requires:

$$\frac{2}{1-\delta} \geq 7 + \frac{5\delta}{1-\delta}.$$

Which yields $\delta \geq \frac{5}{2} > 1$.

More generally, no punishment can exist that is worse for Player 1 than the minimax value. Even using the harshest possible punishment does not help. One can painstakingly compute that the minimax value for Player 1 is approximately $47/13 \approx 3.6 > 2$. Much easier is to note that Player 1 cannot be forced to accept less than 3 in any punishment since O earns at least that for all Player 2's strategies. Hence, the minimax>2 and no SPNE sustains (N, A) on the equilibrium path.

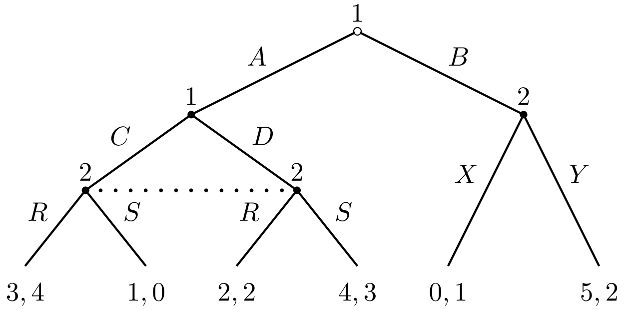
Note 1e.1: Full credit for showing that the unique NE payoff for player 1 is higher than the payoff from (N, A) and thus grim trigger/Nash reversion *rewards* rather than punishes a deviation. Some students invoked the folk theorem, which uses minimax punishments. Player 1's minimax value is $47/13 \approx 3.6 > 2 = u_1(N, A)$, so even the harshest credible punishment cannot make (N, A) incentive-compatible. The impossibility holds under any punishment scheme.

Note 1e.2: Some simply assumed that a grim trigger strategy would support such an equilbrium. First, even when true, such answers depend on the precise values of the cooperative payoffs, "punshment" payoffs, the payoff from deviating from cooperation, and δ.

Note 1e.3: Some argued that this is impossible because the stage game has a unique NE. This confuses the logic of finitely and infinitely repeated games. In an infinitely repeated game, we *can* have many SPNE despite a unique stage-game NE. With the prisoner's dilemma, for example, we can use the infinite future of cooperation as the "reward" and the stage-game NE as the "punishment." Further, the folk theorem tells us that any outcome that does better than NE (or each minimax) can be supported for some range of $\delta < 1$.

Note 1e.4: Some argued that this is impossible because Player 1 has a best response to all Player 2 strategies that give Player 1 a payoff of 7. First, this is only true in response to Player 2's pure strategies. Second, the folk theorem tells us that equilibria that earn less than 7 are possible. In fact, the unique equilibrium of the stage game (in mixed strategies) earns less than 7 which, by this logic, implies that the stage-game equilibrium couldn't be played in any SPNE!

Question 2. Consider the game below. Both the extensive form and the normal form are given. The dotted line represents an information set.



		Player 2			
		R,X	R,Y	S,X	S,Y
Player 1	A,C	3, 4	3, 4	1, 0	1, 0
	A,D	2, 2	2, 2	4, 3	4, 3
	B,C	0, 1	5, 2	0, 1	5, 2
	B,D	0, 1	5, 2	0, 1	5, 2

(a) List all pure-strategy Nash equilibria.

Six pure-strategy NE: $(A,C;\ R,X)$, $(A,D;\ S,X)$, $(B,C;\ R,Y)$, $(B,C;\ S,Y)$, $(B,D;\ R,Y)$, $(B,D;\ S,Y)$.

(b) Which pure-strategy Nash equilibria are trembling-hand perfect? Explain.

THP: $(B,C;\ R,Y)$, $(B,C;\ S,Y)$, $(B,D;\ R,Y)$, $(B,D;\ S,Y)$.

In a 2-player game, a NE is trembling-hand perfect if and only if no player uses a weakly dominated strategy. For Player 2, R,Y weakly dominates R,X and S,Y weakly dominates S,X. The remaining four NE use only R,Y or S,Y for Player 2 and B,C or B,D for Player 1, none of which are weakly dominated. All four are THP.

Note 2b.1: It is certainly okay to show which equilibria are THP from the formal definition, finding a sequence (or showing none exists) that converges to the equilibrium for which the best replies are the same. But this is certainly much more difficult than looking for weakly dominated strategies.

Note 2b.2: Most students correctly noted that a weakly dominated strategy can never be a part of a THP equilibrium. However, it is not true that THP rules out *only* weakly dominated strategies, except in two-player games. Thus, one should be explicit about why the four NE above are THP. It is because they do not involve weakly dominated strategies *and* because this is a two-player game.

(c) List all pure-strategy subgame-perfect Nash equilibria.

Two pure-strategy SPNE: $(B,C;\ R,Y)$ and $(B,D;\ S,Y)$.

There are two proper subgames:

After B: Y is the unique NE.

After A: The subgame is a 2×2 game (Player 1 chooses C or D, Player 2 chooses R or S). Pure NE of this subgame: (C,R) and (D,S).

Therefore, any Nash equilibrium that involves Y and either (C,R) or (D,S) is subgame perfect

Same answer can come from backward induction by noting that against either subgame NE after A, Player 1 gets 3 or 4, so Player 1 alwas prefers B with a payoff 5.

(d) Consider the best Nash equilibrium from (a) for Player 2. Carefully explain why it is a Nash equilibrium but is not a subgame-perfect Nash equilibrium.

Best NE for Player 2: $(A, C;\ R, X)$ with payoff 4.

Why it is a NE: Against R, X, Player 1's best reply is A, C (payoff 3, the highest in column R, X). Against A, C, Player 2's best replies are R, X and R, Y (both give 4). Neither player has a strictly profitable deviation.

Why it is not a SPNE: In the subgame after B, Player 2's strategy prescribes X (since the strategy is ‘‘R, X’’). But in that subgame Y strictly dominates X (payoffs 2 vs. 1). Player 2 is not playing a Nash equilibrium of the subgame after B. The threat to play X following B deters Player 1 from deviating (playing B would yield 0), but the threat is not credible: if B were reached, Player 2 would rationally switch to Y.

Intuitive difference: In response to A, the choice of action X or action Y is payoff irrelevant as it is off the equilibrium path. Nash equilibrium does not put constraints on such off-equilibrium actions (since they are all a best reply since they will never be payoff relevant). SPNE does constrain them to be best replies conditional on reaching the subgame.

Note 2d.1: The reason why this is not an SPNE is that Player 2 would never play X in the right subgame. It is not that Player 1 is playing A, since that is the best choice in the relevant subgame (the game as a whole) *given* Player 2 chooses X.

(e) Imagine that the information set following actions C and D did not exist (i.e., the game is one of perfect information, with each information set containing a single node). Identify all subgame-perfect Nash equilibria of this modified game.

Unique SPNE: $\{B, D; R$ (at C), S (at D), $Y\}$.

Backward induction with singleton information sets:

- After B: Player 2 prefers Y (2 vs. 1). Play Y.
- At the C-node: Player 2 sees C was played. Prefers R (4 vs. 0). Play R.
- At the D-node: Player 2 sees D was played. Prefers S (3 vs. 2). Play S.
- Player 1's second move (after A): C (then R) yields payoff 3, D (then S) yields payoff 4. Play D.
- Player 1's first move: A yields payoff 4; B yields payoff 5. Play B.

No ties in backward induction, so the SPNE is unique.

Note 2e.1: In the original game, the information set prevents Player 2 from conditioning on C vs. D. Removing it lets Player 2 best-respond to each separately. This means that a strategy for Player 2 now specifies three actions rather than two. That is, Player 2's strategy space in the original game is not the same as in the modified game.

Note 2e.2: This is a game of perfect information and there are no “ties” anywhere, so the equilibrium must be unique and in pure strategies.

Question 3. The U.S. government is considering subsidies to support domestic investment in semiconductor R&D. First, the government chooses a government support level $g \geq 0$ at cost g^2. Next, two firms observe g and simultaneously choose R&D investment levels $r_1, r_2 \geq 0$. Firm i's profit (when the other firm is firm j) is:

$$\pi_i(g, r_i, r_j) = \sqrt{g + r_j}\,\sqrt{r_i} - r_i$$

The government's objective is to maximize total firm investment minus the cost of government support: $U_G = r_1 + r_2 - g^2$.

(a) What is the subgame-perfect Nash equilibrium?

$\boxed{\text{SPNE: } g^* = \tfrac{1}{3};\ r_1^* = r_2^* = \tfrac{1}{3}g.}$

Stage 2 (firms' simultaneous choice of r_i, given g):

Firm i maximizes $\pi_i = \sqrt{g + r_j} \cdot \sqrt{r_i} - r_i$. FOC:

$$\frac{\partial \pi_i}{\partial r_i} = \frac{\sqrt{g + r_j}}{2\sqrt{r_i}} - 1 = 0 \implies \sqrt{r_i} = \tfrac{1}{2}\sqrt{g + r_j} \implies \boxed{r_i(r_j) = \tfrac{g + r_j}{4}}.$$

This is firm i's best response function. Substituting,

$$r_i = \tfrac{g + r_j}{4} = \frac{g + \frac{g + r_i}{4}}{4} = \frac{5}{16}g + \frac{1}{16}r_i \implies \boxed{r_i^*(g) = \frac{1}{3}g}.$$

This is firm i's equilibrium strategy. Each firm invests $g/3$ in stage 2.

Stage 1 (government chooses g):

The government maximizes $U_G = r_1^* + r_2^* - g^2 = \frac{2}{3}g - g^2$. FOC: $\frac{dU_G}{dg} = \frac{2}{3} - 2g = 0 \implies \boxed{g^* = \frac{1}{3}}$.

Note 3a.1: Symmetric objective functions imply symmetric best reply functions. They do not imply symmetric equilibria or uniqueness of a symmetric equilibrium unless additional conditions (e.g., convexity of utility, compactness of strategy space) are satisfied.

Note 3a.2: Recall that a strategy for the firms specifies an r_i for each g, and is therefore a function $r_i(g) = \frac{1}{3}g$, not a single number, $\frac{1}{9}$, which represents the equilibrium outcome given equilibrium g^*.

(b) What is the government's equilibrium payoff?

$$U_G^* = r_1^* + r_2^* - (g^*)^2 = \tfrac{1}{9} + \tfrac{1}{9} - \left(\tfrac{1}{3}\right)^2 = \tfrac{2}{9} - \tfrac{1}{9} = \boxed{\frac{1}{9}}.$$