# Microeconomic Theory II — Spring 2025

# Midterm Exam — Mikhael Shor

## Solutions
Quick, Incomplete, and not Guaranteed

**Question 1.** Consider the following game.

| | | Player 2 | | | |
|---|---|---|---|---|---|
| | | $A$ | $B$ | $C$ | $D$ |
| Player 1 | $M$ | $3,8$ | $1,20$ | $2,1$ | $2,6$ |
| | $N$ | $5,0$ | $2,1$ | $1,2$ | $1,1$ |
| | $O$ | $3,5$ | $3,1$ | $5,3$ | $8,0$ |
| | $P$ | $2,1$ | $4,5$ | $4,3$ | $4,100$ |

(a) What strategies are consistent with rationality? Carefully explain why each strategy is or is not.

`{ N,O,P ; A,B,C,D }`

We must show for each strategy either that it is a best response to some beliefs or that it is not strictly dominated (these are the same in two-player games). All of the above are best replies to a pure strategy, leaving only M to check. M is strictly dominated by (for example) $\frac{1}{2}N + \frac{1}{2}O$ and therefore is not consistent with rationality.

**Note 1.1** : It is not sufficient to show that M is not a best reply to any pure strategy of Player 2. One must show that it is not a best reply to *any* strategy of player 2 (or alternatively, that it is strictly dominated).

(b) What strategies survive the iterated deletion of strictly dominated strategies? For each iteration, specify the dominated strategy and a strategy that dominates it.

`{ N,O ; A,C }`

M is strictly dominated by (for example) $\frac{1}{2}N + \frac{1}{2}O$
B is strictly dominated by $\frac{1}{2}C + \frac{1}{2}D$
P is strictly dominated by O
D is strictly dominated by C
none of the remaining strategies is dominated.

**Note 1.2** : Some students calculated all mixed strategies that dominate a particular strategy. While this is certainly fine, recall that you need only show some strategy that dominates it (as in the answer above).

(c) List all Nash equilibria of this game.

$\boxed{\{\frac{1}{2}N + \frac{1}{2}O; \frac{2}{3}A + \frac{1}{3}C\}}$
This is the unique NE.

(d) What are each player's expected equilibrium payoffs?

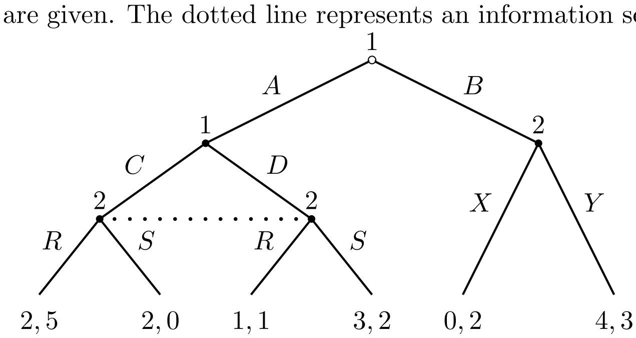
$\boxed{\frac{11}{3} \text{ and } \frac{5}{2}}$.

(e) Imagine that the above (stage) game is repeated twice, with players observing the outcome of the first stage before playing in the second stage. Is there any subgame-perfect Nash equilibrium in which either player earns strictly more in the first period than the payoffs calculated above? Briefly explain.

$\boxed{\text{No}}$
Since the stage game has a unique equilibrium, any finite repetition of the stage game will have a unique SPNE involving playing the stage game equilibrium at each stage.

**Note 1.3** : Some students confused the answer above with the fact that SPNE $\subseteq$ NE. The latter is true but don't confuse the Nash equilibrium of the stage game with the Nash equilibria of the repeated game. The repeated game has many Nash equilibria but only one SPNE.

**Question 2.** Consider the game below. Both the extensive form and the normal form are given. The dotted line represents an information set.




Player 2

| Player 1 | | $R, X$ | $R, Y$ | $S, X$ | $S, Y$ |
|---|---|---|---|---|---|
| | $A, C$ | 2, 5 | 2, 5 | 2, 0 | 2, 0 |
| | $A, D$ | 1, 1 | 1, 1 | 3, 2 | 3, 2 |
| | $B, C$ | 0, 2 | 4, 3 | 0, 2 | 4, 3 |
| | $B, D$ | 0, 2 | 4, 3 | 0, 2 | 4, 3 |

(a) List all pure-strategy Nash equilibria.

$\{A, C; R, X\}, \{A, D; S, X\}, \{B, C; R, Y\}, \{B, C; S, Y\}, \{B, D; R, Y\}, \{B, D; S, Y\}$

(b) Which pure-strategy Nash equilibria are trembling-hand perfect? Explain.

$\{B, C; R, Y\}, \{B, C; S, Y\}, \{B, D; R, Y\}, \{B, D; S, Y\}$

In two-player games, THPE are all NE that do not involve weakly dominated strategies. $R, X$ is weakly dominated by $R, Y$ and $S, X$ is weakly dominated by $S, Y$.

**Note 2.1** : Recall that every finite game has a THPE.

(c) List all pure-strategy subgame-perfect Nash equilibria.

$\{B, C; R, Y\}, \{B, D; S, Y\}$

Pure strategy equilibria of the left subgame are $\{C, R\}$ and $\{D, S\}$ and equilibrium of the right subgame is $Y$.

(d) Consider the best Nash equilibrium from (a) for Player 2. Carefully explain why it is a Nash equilibrium but is not a subgame-perfect Nash equilibrium.

SPNE requires Nash equilibrium play in every subgame, even ones unreached in equilibrium, while NE does not restrict play "off the equilibrium path." The Nash equilibrium $\{A, C; R, X\}$ involves player 2 playing $X$ in case of $B$, but if $B$ were actually played, in the resulting subgame, player 2 would play $Y$.

**Question 3.** The owner of a firm can invest in technology that improves the productivity of his two employees. The timing of the game is:

- The owner selects a level of technology, $\tau$, at a cost of $\tau^3$, then
- Both employees, after observing $\tau$, simultaneously select their effort levels $e_1 \geq 0$ and $e_2 \geq 0$ at a cost of $\frac{1}{4}e_i^2$.

The total revenue of the firm, as a function of $\tau, e_1, e_2$ is given by:

$$R = \tau e_1 + \tau e_2 + e_1 e_2$$

The revenue of the firm is shared with the employees, with $\frac{1}{2}$ going to the owner and $\frac{1}{4}$ to each employee. Thus, the owner's profit is $\frac{1}{2}R - \tau^3$ and the utility of employee $i$ is $\frac{1}{4}R - \frac{1}{4}e_i^2,\ i \in \{1, 2\}$.

(a) Determine the subgame-perfect Nash equilibrium of this game.

$\boxed{\{\tau = 1; e_1(\tau) = e_2(\tau) = \tau\}}$

`We begin in the second period. Employee` $i$`'s best response is given by` $e_i = \frac{1}{2}(\tau + e_j)$ `which yields a NE of` $e_1(\tau) = e_2(\tau) = \tau$.
`In the first period, the owner's profit is` $\frac{1}{2}(\tau e_1(\tau) + \tau e_2(\tau) + e_1(\tau)e_2(\tau)) - \tau^3$ `which equals (substituting second-period equilibrium)` $\frac{3}{2}\tau^2 - \tau^3$. `Maximizing yields` $\tau = 1$.

**Note 3.1** : Be sure not to confuse $e_i = \tau$ which is an employee's SPNE strategy (specifying an action for every subgame, $\tau$) with the equilibrium path $e_i = 1$ for the equilibrium value of $\tau$.

**Note 3.2** : The fact that the payoff functions for two players are identical does not mean that all (or any) Nash equilibria will be symmetric (though the best responses, obviously, will be). Thus, one cannot assume in a solution that $e_1 = e_2$ but must obtain this from the best responses.

(b) Imagine that the owner decides to share more revenue with the employees, with all three (the owner and the employees) receiving $\frac{1}{3}R$. Does the owner's profit in equilibrium increase or decrease? Demonstrate or explain.

`The second period equilibrium is now` $e_1(\tau) = e_2(\tau) = 2\tau$. `Substituting into the owner's profit yields` $\frac{8}{3}\tau^2 - \tau^3$. `Observe that this is higher for` *every* $\tau$ `than the owner's profit in part (a) so profit clearly increases with no further calculations necessary (though the optimal` $\tau$ `also increases from 1 to` $\frac{16}{9}$`).`

**Note 3.3** : Interestingly, in this problem, the owner's profit is increasing in the share given to each employee for all shares less than $\frac{1}{2}$. The second-period equilibrium as a function of $\tau$ and of share to each employee, $s$, is

$e(s,\tau) = \left(\frac{2s}{1-2s}\right)\tau$. Substituting into the owner's profit (who receives share $1-2s$):

$$\pi(\tau, s) = \frac{4s(1-s)}{1-2s}\tau^2 - \tau^3$$

which is increasing in $0 \leq s < \frac{1}{2}$ when $\tau > 0$. Therefore, the extra effort induced by an increase in employee share increases the owner's profit even without a change in $\tau$!

Further, optimal $\tau^*(s) = \frac{8s(1-s)}{3(1-2s)} \to \infty$ as $s \to \frac{1}{2}^-$,

and $\pi(\tau^*(s), s) = \frac{256s(1-s)^3}{9(1-2s)^3} \to \infty$ as $s \to \frac{1}{2}^-$.

So, ironically, as the owner's share goes to 0, the owner's profit goes to infinity.