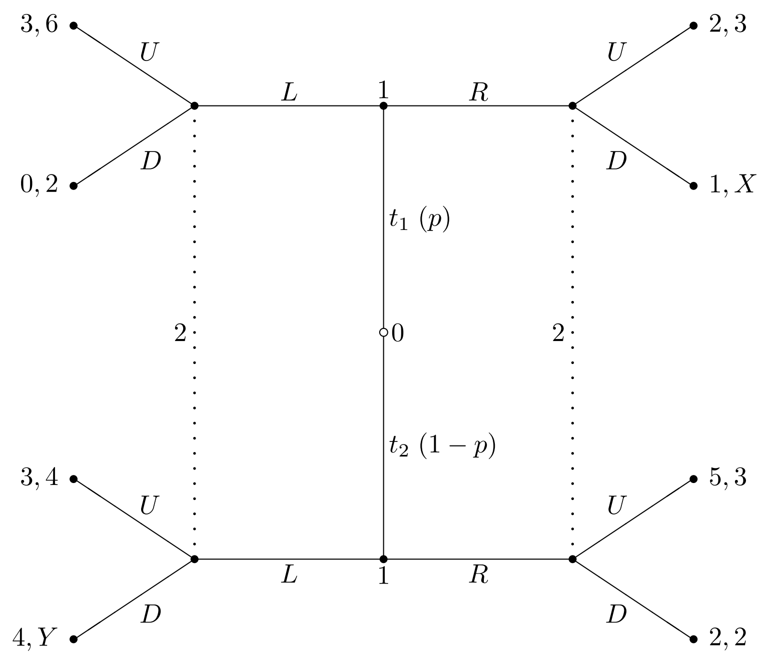
Microeconomic Theory II — Spring 2024

Final Exam — Mikhael Shor

Carefully explain and support your answers.

Question 1. Consider the following game. First, nature (player 0) selects t_1 with probability p, $0 < p < 1$, or t_2 with probability $1-p$. Next, player 1 selects L or R. Lastly, player 2 selects U or D.



(a) Assume $X = Y = 5$. Find all separating equilibria.

(b) Assume $p = \frac{1}{2}$. Find all values of X and Y for which there exists a pooling equilibrium on R. Carefully explain.

(c) Assume $p = \frac{1}{2}$. Find all values of X and Y for which there exists a pooling equilibrium on L. Carefully explain.

(d) Does the pooling equilibrium on L derived in the previous part satisfy the intuitive criterion? Carefully explain.

Question 2. Consider a principal-agent model in which the agent chooses between two levels of effort, $\{e_l, e_h\}$. The principal pays the agent a wage w_s in state s and realizes output of π_s. There are four states, with $(\pi_1, \pi_2, \pi_3, \pi 4) = (500, 430, 20, 0)$ and the probability of a state contingent on effort given by:

effort level	π_1	π_2	π_3	π_4
e_h	.3	.3	.3	.1
e_l	.1	.1	.6	.2

The agent's utility function is $\sqrt{w} - c(e)$ where $c(e_h) = 4, c(e_l) = 0$, and his reservation utility is $\underline{u} = 1$. The principal is risk neutral, with utility given by $\pi - w$.
Wages may not be negative.

(a) Determine the wage schedule that optimally implements e_h when effort is observable.

(b) Determine the wage schedule that optimally implements e_h when effort is unobservable.

(c) When effort is unobservable, what effort level does the principal wish to implement?

(d) Imagine that the government institutes a minimum wage, $\hat{w}$, requiring that $w(\pi) \geq \hat{w} \ \forall \pi$. Assume that effort is unobservable. Show that the principal is indifferent between implementing high effort and low effort when $\hat{w} = 100$.

Question 3. Two identical firms (1 and 2) produce a homogeneous product. Competition takes place over two periods. In the first period, each firm simultaneously selects a level of advertisement, $a_i \geq 0, i \in \{1, 2\}$, with costs given by $c(a_i) = \frac{1}{3}a_i^2$. In the second period, after observing first-period advertising expenditures, each firm selects a quantity, q_i. The industry inverse demand function is given by $p = a_1 + a_2 - q_1 - q_2$. Marginal costs of production are zero. Each firm maximizes profit, given by $pq_i - c(a_i)$.

(a) What is the subgame perfect equilibrium? Carefully show all work.

(b) What are each firm's equilibrium profits?

(c) Imagine that each firm set its level of advertising to 1 ($a_1 = a_2 = 1$) in the first period, and then played its Nash equilibrium strategy in the second period. What would be each firm's profit?

(d) Does there exist a Nash equilibrium in which both firms set their levels of advertising to 1? Briefly explain why or why not.