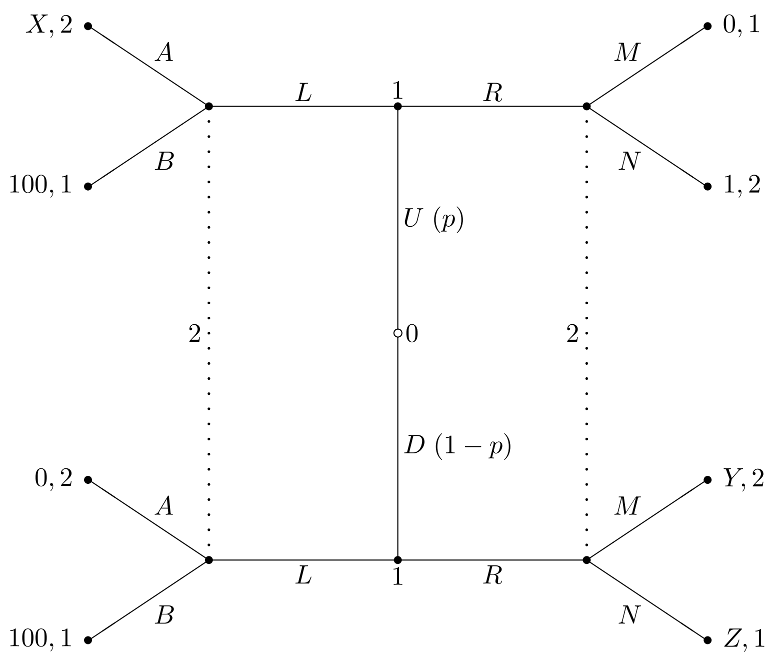
Microeconomic Theory II — Spring 2023

Final Exam Solutions — Mikhael Shor

Question 1. Consider the following game. First, nature (player 0) selects U with probability p or D with probability $1-p$. Next, player 1 selects L or R. Lastly, player 2 selects either A or B (if player 1 selected L) or M or N (if player 1 selected R).



Assume throughout that $p < \frac{1}{2}$.

(a) What are each player's pure strategies?
`For the sender,` $\{LL, LR, RL, RR\}$`; for the receiver,` $\{AM, AN, BM, BN\}$

(b) Assume $X = Y = Z = 2$ and recall that $p < \frac{1}{2}$. Find all pure-strategy weak perfect Bayesian equilibria (and show or explain that none other exist).
`First, note that the receiver will always select` A `for any beliefs. This implies that type` U `will select` L `(since` $2 > 0$ `and` $2 > 1$`) and type` D `will select` R `(`$2 > 0$`). In response to only` D `selecting` R`, the receiver selects` M`. Thus,` $\{LR, AM\}$ `with degenerate beliefs is the unique wPBNE.`

(c) Find all values of X, Y, and Z such that *both* types of pooling equilibria (LL and RR) exist. Carefully demonstrate or explain.

Here, I consider only pure-strategy pooling equilibria. Start with pooling on L. The receiver picks A. To prevent deviations by the sender, we need either ($X \geq 0$ and $0 \geq Y$) if beliefs were such that the receiver selects M or ($X \geq 1$ and $0 \geq Z$) if the receiver selects N.

Next, consider pooling on R. Given that $p < \frac{1}{2}$, the receiver selects M. For any beliefs, the receiver selects A. To prevent deviations by the sender, we need $0 \geq X$ and $Y \geq 0$.

Combining the sets of conditions for each pooling equilibrium gives: $X = 0, Y = 0$ with no restrictions on Z.

(d) Find all values of X, Y, and Z such that *both* types of separating equilibria (LR and RL) exist. Carefully demonstrate or explain.

First, consider $U \rightarrow L, D \rightarrow R$. The receiver's best response is A, M. To keep U from deviating, we need $X \geq 0$. To keep D from deviating, we need $Y \geq 0$.

Next, consider $U \rightarrow R, D \rightarrow L$. The receiver's best response is A, N. To keep U from deviating, we need $X \leq 1$. To keep D from deviating, we need $Z \leq 0$.

Combining the above conditions, $0 \leq X \leq 1, Y \geq 0, Z \leq 0$.

Question 2. Consider a principal-agent model in which the agent has two levels of effort, $e \in \{L, H\}$. There are four different outcomes associated with different profits for the principal, $(\pi_1, \pi_2, \pi_3, \pi_4)$. Define p_i^e as the probability of outcome i when level of effort is e.

The principal is risk neutral with utility given by profits minus wages. The agent's utility function is (of course) given by $u(w, e) = \sqrt{w} - c(e)$.

The cost to the agent of the two types of effort are $c(L) = 14, c(H) = 20$. Reservation utility is 0.

		outcome 1	outcome 2	outcome 3	outcome 4
$(p_1^L, p_2^L, p_3^L, p_4^L)$	=	1/20	2/20	8/20	9/20
$(p_1^H, p_2^H, p_3^H, p_4^H)$	=	3/20	6/20	8/20	3/20

Wages cannot be negative (you may assume that these constraints never bind, however).

(a) If effort can be observed, what is the optimal contract for inducing *low* effort?

`we need` $\sqrt{w} = 14$ `or` $w^L = 196$

(b) If effort can be observed, what is the optimal contract for inducing *high* effort?

`we need` $\sqrt{w} = 20$ `or` $w^H = 400$

(c) Assume that effort cannot be observed (but outcomes can). Derive the optimal contract for inducing *low* effort.

`We pay a fixed wage as above,` $w_1 = w_2 = w_3 = w_4 = 196$

(d) Assume that effort cannot be observed (but outcomes can). Derive the optimal contract for inducing *high* effort. Carefully identify all constraints. [Sizable hint: No derivatives are necessary]

`Note first that` $\frac{p_1^L}{p_1^H} = \frac{p_2^L}{p_2^H}$ `so it must be that` $w_1 = w_2$. `Also note that outcome 3 is not informative, so (since the non-negativity constraints don't bind), the wage is the same as in the observable case,` $w_3 = 400$.

`The IC constraint is:`

$$\frac{3}{20}\sqrt{w_1}+\frac{6}{20}\sqrt{w_2}+\frac{8}{20}\sqrt{w_3}+\frac{3}{20}\sqrt{w_4}-20 \geq \frac{1}{20}\sqrt{w_1}+\frac{2}{20}\sqrt{w_2}+\frac{8}{20}\sqrt{w_3}+\frac{3}{20}\sqrt{w_4}-14$$

`which must bind, and reduces to:`

$$\sqrt{w_1} = 20 + \sqrt{w_4})$$

`The IR constraint is:`

$$\frac{3}{20}\sqrt{w_1}+\frac{6}{20}\sqrt{w_2}+\frac{8}{20}\sqrt{w_3}+\frac{3}{20}\sqrt{w_4}-20\geq 0$$

which becomes

$$\frac{9}{20}\sqrt{w_4}+9+8+\frac{3}{20}\sqrt{w_4}-20\geq 0$$

which must bind, yielding $\sqrt{w_4}=5$

Therefore $\sqrt{w_1}=\sqrt{w_2}=25, \sqrt{w3}=20, \sqrt{w4}=5$ or $w_1=w_2=625, w_3=400, 2_4=25$.

(e) If the principal wants to induce high effort, how much higher are average wages when effort is not observable than when effort is observable?

`Expected wages under high effort are:`

$$\frac{9}{20}625+\frac{8}{20}400+\frac{3}{20}25=445$$

`which is` 45 `higher than the payment under perfect information.`

Question 3. Northwestern Connecticut University (NW) competes with Southeastern Connecticut University (SE) for students wanting to be ready for the latest high-tech jobs. Each is deciding whether to open either an Institute of Data Science or an Institute of Crypto. Data Science is a larger market. Specifically, (inverse) demand for data science is given by:

$$p_D = 60,000 - 2Q_D$$

where p_D is the tuition charged and Q_D is the total enrollment in data science across all schools that open an Institute of Data Science.

Similarly, (inverse) demand for crypto is given by:

$$p_C = 30,000 - Q_C$$

where p_C is the tuition charged and Q_C is the total enrollment in crypto across all schools that open an Institute of Crypto.

The interaction proceeds over two years. In year one, each school simultaneously selects $I \in \{D, C\}$ (whether to create an Institute of Data Science or an Institute of Crypto). In year 2, after observing each other's institute choices, each selects the size of its enrollment, q_I. Finally, a school's profit is given by $q_I p_I$, its enrollment times the tuition for I.

1. Identify all pure-strategy subgame-perfect Nash equilibria.

   Note that the profit in D is simply twice the profit in C so the best replies in both markets are the same:

$$q_i = 1500 - \frac{1}{2}q_j$$

   When q_j picks a different institute, $q_i* = 15000$. When they pick the same institute, $q_i = q_j = 10000$. Solving for profit (dividing q and p by 1000), we get:

   - Both choose D: $10 \times 20 = 200$
   - Both choose C: $10 \times 10 = 100$
   - One chooses D: $15 \times 30 = 450$
   - One chooses C: $15 \times 15 = 225$

	C	D
C	$100, 100$	$225, 450$
D	$450, 225$	$200, 200$

   The stage game has two pure-strategy equilibria.

   So, the SPNE are:

$$\{C, 10000, 10000, 15000, 15000; D, 10000, 10000, 15000, 15000\}, \text{and}$$

$$\{D, 10000, 10000, 15000, 15000; C, 10000, 10000, 15000, 15000\},$$

   where the numbers reflect the quantity choices in each of the four proper subgames.

2. Imagine that the decision to announce an institute is also sequential. Would a university prefer to announce first or second? Briefly explain.

   `Announcing first, a university would pick` D `with the other university responding with` C`. Thus, the first mover earns` 450 `and the second earns` 225`. A university would prefer to announce first.`