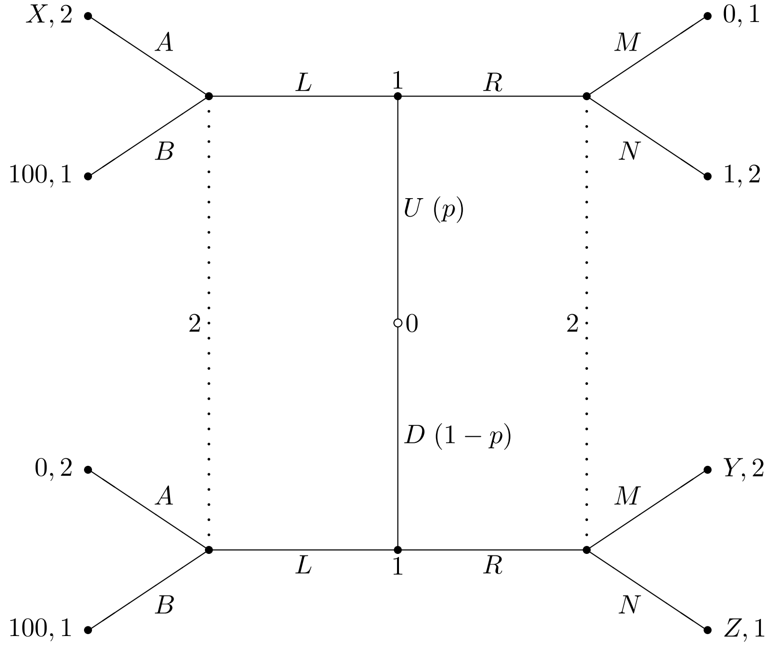
Microeconomic Theory II — Spring 2023

Final Exam — Mikhael Shor

Carefully explain and support your answers.

Question 1. Consider the following game. First, nature (player 0) selects U with probability p or D with probability $1-p$. Next, player 1 selects L or R. Lastly, player 2 selects either A or B (if player 1 selected L) or M or N (if player 1 selected R).



Assume throughout that $p < \frac{1}{2}$.

(a) What are each player's pure strategies?

(b) Assume $X = Y = Z = 2$ and recall that $p < \frac{1}{2}$. Find all pure-strategy weak perfect Bayesian equilibria (and show or explain that none other exist).

(c) Find all values of X, Y, and Z such that *both* types of pooling equilibria (LL and RR) exist. Carefully demonstrate or explain.

(d) Find all values of X, Y, and Z such that *both* types of separating equilibria (LR and RL) exist. Carefully demonstrate or explain.

Question 2. Consider a principal-agent model in which the agent has two levels of effort, $e \in \{L, H\}$. There are four different outcomes associated with different profits for the principal, $(\pi_1, \pi_2, \pi_3, \pi_4)$. Define p_i^e as the probability of outcome i when level of effort is e.

The principal is risk neutral with utility given by profits minus wages. The agent's utility function is (of course) given by $u(w, e) = \sqrt{w} - c(e)$.

The cost to the agent of the two types of effort are $c(L) = 14, c(H) = 20$. Reservation utility is 0.

		outcome 1	outcome 2	outcome 3	outcome 4
$(p_1^L, p_2^L, p_3^L, p_4^L)$	=	1/20	2/20	8/20	9/20
$(p_1^H, p_2^H, p_3^H, p_4^H)$	=	3/20	6/20	8/20	3/20

Wages cannot be negative (you may assume that these constraints never bind, however).

(a) If effort can be observed, what is the optimal contract for inducing *low* effort?

(b) If effort can be observed, what is the optimal contract for inducing *high* effort?

(c) Assume that effort cannot be observed (but outcomes can). Derive the optimal contract for inducing *low* effort.

(d) Assume that effort cannot be observed (but outcomes can). Derive the optimal contract for inducing *high* effort. Carefully identify all constraints. [Sizable hint: No derivatives are necessary]

(e) If the principal wants to induce high effort, how much higher are average wages when effort is not observable than when effort is observable?

Question 3. Northwestern Connecticut University (NW) competes with Southeastern Connecticut University (SE) for students wanting to be ready for the latest high-tech jobs. Each is deciding whether to open either an Institute of Data Science or an Institute of Crypto. Data Science is a larger market. Specifically, (inverse) demand for data science is given by:

$$p_D = 60,000 - 2Q_D$$

where p_D is the tuition charged and Q_D is the total enrollment in data science across all schools that open an Institute of Data Science.

Similarly, (inverse) demand for crypto is given by:

$$p_C = 30,000 - Q_C$$

where p_C is the tuition charged and Q_C is the total enrollment in crypto across all schools that open an Institute of Crypto.

The interaction proceeds over two years. In year one, each school simultaneously selects $I \in \{D, C\}$ (whether to create an Institute of Data Science or an Institute of Crypto). In year 2, after observing each other's institute choices, each selects the size of its enrollment, q_I. Finally, a school's profit is given by $q_I p_I$, its enrollment times the tuition for I.

1. Identify all pure-strategy subgame-perfect Nash equilibria.
2. Imagine that the decision to announce an institute is also sequential. Would a university prefer to announce first or second? Briefly explain.